Closing the Loop of Sound Evaluation and Design (CLOSED)

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Algorithms for Ecologically-Founded Sound Synthesis: Library and Documentation

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Physically-based

Sound Design Tools

User’s Guide
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This is a trick...
Introduction

The Physically-based Sound Design Tools (SDT from now on) aims at providing perception-oriented and physically-coherent advanced tools for the next generation of sound designers.

The SDT package is the main software product of a project activity which begun in 2001 with the EU project SOb - the Sounding Object [3]. In continuous development, the SDT package inherits basic theoretical concepts of sound synthesis initiated with the physical analysis of impact sounds, later complemented with more low-level physically-based sound models (such as friction) and further provided with higher level synthesis paradigms, integrating low-level models into more elaborate synthesis contexts (such as rolling and crumpling).

Such theoretical concepts found early realizations in the form of pure data patches and externals for Linux and Windows which had to be ad hoc compiled. In SDT all externals have been thoroughly revised and rewritten to comply with the multi-OS programming layer flext and to run in the Max/MSP environment. Moreover, the adoption of Max/MSP has allowed to restyle the early pure data patches in order to create more accessible, usable sound synthesis controls and better online documentations.

The SDT package is maintained and continuously developed by the UNIVERONA (VIPS group) unit of the EU project CLOSED - Closing the Loop Of Sound Evaluation and Design.

The basic (low-level) algorithms, the corresponding externals and some example patches are described in Chapter 1.

Higher level algorithms, based on the basic ones, are described in Chapter 2. In some cases the higher level algorithms are just implemented as Max/MSP patches which exploit the basic externals; in others they rely on both low-level and additional externals.

Externals descriptions are easily identifiable by boxed titles.

Throughout this manual we follow the Mac OS X (or UNIX) file-system conventions: e.g. paths are expressed with slashes “/”. Of course Windows users just need to consider back-slashes “\” instead of slashes.

How to use the SDT package

The SDT package is split into three main sections (corresponding to subdirectories): source code (under /SDT_sources), Max/MSP patches (under /SDT_patches) and, for the faint hearted, pre-compiled Max/MSP externals for both Mac OS X [1] and Windows (under /SDT_binaries).

All main source files are coded in C++ (while some other auxiliary files are coded in C) using flext a layer for cross-platform development of Max/MSP and pure data externals. This means that, with minimum effort, one can build the SDT package on Mac OS X, Windows and Linux platforms, this way getting externals for Max/MSP or pure data. In other words, although this manual is oriented towards Max/MSP, still sound designers keen on pure data can build the very same externals for their application of choice. Unfortunately though, the SDT package only include patches for Max/MSP.

Building flext

Despite the provided Max/MSP externals being ready for use, dauntless users would surely like to build their own ones. Other users may want to modify some of the provided externals, therefore

1Mac OS X externals are in universal binary format, that is they are compatible with both PPC and Intel Macs

http://grrrr.org/ext/flext/
they would need to alter the corresponding source code, and then build their updated externals from scratch.

For all these advanced users the first thing to do is to download and install the **Max/MSP Software Development Kit (SDK)** from [http://www.cycling74.com/downloads/maxmsp](http://www.cycling74.com/downloads/maxmsp).

Secondly, it is necessary to download and install `flext`. At the time of writing, the version of `flext` required to correctly build the SDT package is only available as CVS check-out (starting with the current v0.5.1, any bleeding edge version should fit).

If your OS is Windows and you are not using cygwin, make sure that the option “use UNIX line endings” (or the like) of your CVS client is checked. If you don’t know how to do that, and your client does not do it by default, after having checked-out `flext` you’d most likely need to open all text-type files lying inside the directory `/flext` and its subdirectories, change all line endings to UNIX style, and save them.

Here are all the informations needed in order to check-out the CVS version of `flext` with any CVS client:

**Protocol:** :pserver:

**User name:** anonymous

**Server:** pure-data.cvs.sourceforge.net

**Repository directory:** /cvsroot/pure-data

**Module:** externals/grill/flext

Once the check-out is finished, it is necessary to configure, build and install `flext` on your machine. Please refer to the files readme.txt and build.txt inside the directory `/flext` of your `flext` distribution for detailed instructions on how to do that.

**Building the SDT package**

All source files lie inside the directory `/SDT_sources` (under `/Solids` and `/Liquids`).

To each external corresponds a `.txt` file containing instruction for the compiler and being used by `flext` own build system. In `flext`-based distributions, the default name for this kind of file is `package.txt` so that, when invoking the build batch-file, you don’t have to specify its name. However, since SDT is made up of many externals which often share the same source files, we opted for putting all shared sources together, providing a `EXTERNAL_NAME.txt` file for each external. Hence, when invoking `flext` build batch-file it is necessary to add the macro ‘‘PKGINFO=EXTERNAL_NAME.txt’’ (including quotation marks) at the end of your sequence of instructions.

**Hint 1:** In order to refer to them more quickly, you can rename each package-type `.txt` file as you wish, e.g. with progressive numbers: 1.txt, 2.txt, etc..

**Hint 2:** The easiest way to compile the externals is to copy the whole directory `/SDT_sources` under the directory `/flext` of your `flext` distribution. Then it will be straightforward to invoke `flext` build batch-file as:

```
sh ../build.sh
```

from bash shell, or

```
..\build
```

3With a suitable text editor like the free Notepad++ [http://notepad-plus.sourceforge.net/]
from DOS command prompt.

**Example 1:** To build the external `impact_2modalb~` with gcc under Mac OS X, just open the terminal, go inside `/SDT_sources` and write:

```
sh ../build.sh max gcc "PKGINFO=impact_2modalb~.txt"
```

or change `../build.sh` accordingly to the specific path of your flex distribution. Then you'll find the compiled external within the directory `/max-darwin` under `/SDT_sources`.

**Example 2:** To build the external `onebubble~` with Microsoft Visual Studio under Windows, open the Visual Studio Command Prompt, go inside `\SDT_sources` and write:

```
..\build max msvc "PKGINFO=onebubble~.txt"
```

or change `..\build` accordingly to the specific path of your flex distribution. Then you'll find the compiled external within the directory `\max-msvc` under `\SDT_sources`.

**Installing the SDT package**

To install the SDT package:

1. Copy all the externals for your OS into Max/MSP `/externals` directory or, provided that you update accordingly Max/MSP preferences, into your directory of choice.

2. Copy all `.help` files found under `/SDT_patches` and subdirectories into Max/MSP `/max-help` directory.

3. Start using the SDT by double-clicking on any `.mxb` example patch under `/SDT_patches` and have fun!
1 Low-level models

Basic models for solids-contact and liquids sound events are presented. After some theory, externals which implement the models and some example patches are described.

We will see in Chapter 2 how these basic models serve as a basis for more complex sound events, textures and processes.

1.1 Solids contact

The models considered here apply to basic contact events between two solid objects. As the most relevant contact sound events in everyday life come down to impacts and frictions, the provided externals model these two kinds of interactions.

The algorithms implemented here share a common structure: two solid object models interact through (what we call) an interactor (see Fig. 1.1).

Contact models

An interactor represents a contact model or, so to say, the “thing” between the two interacting objects. As for the impact model, it can be seen as the “felt” between the striking object and the struck object, while in the friction model it simulates friction as if the surfaces of the two rubbing objects would be covered with “micro-bristles”.

Two impact models (one of which is a simplified version) and a friction model are provided:

impact interactor - implements a non-linear impact force. It receives the total compression (the difference of displacements of the two interacting objects at interaction point) and returns the computed impact force.

The latter is made of the sum of an elastic component and a dissipative one. The elastic component is parameterized by the force stiffness (or elasticity) and a non-linear exponent.
which depends on the local geometry around the contact area. The dissipative component is parameterized by the force dissipation (or damping weight). For further details refer to [3].

**simplified impact interactor** - implements a linear impact force. As it is a linearized version of the impact interactor described above, the non-linear exponent is not present (or, equivalently, is equal to 1), while the force stiffness and the force dissipation still remain.

**friction interactor** - implements a non-linear friction force. It receives the relative velocity of the two rubbing objects and returns the computed friction force. The latter is made of the sum of four components, each of them corresponding to one coefficient. These are: an elasticity coefficient, an internal dissipation coefficient, a viscosity coefficient, and finally the gain of a pseudo-random function (noise related to surface roughness). The model is parametrized by several others quantities: the dynamic friction and the static friction coefficients, a break-away coefficient and the Stribeck velocity (both of them relate to the transient). For further details refer to [3].

**Object models**

Three distinct object models are provided:

**modal object** - in the modal description, a resonating object is described as a system of a finite number of parallel mass-spring-damper structures. Each mass-spring-damper structure models a mechanical oscillator which represents a normal mode of resonance of the object. The oscillation period, the mass and the damping coefficient of each oscillator correspond respectively to the resonance frequency, the gain and the decay time of each mode. In our implementation it is possible to choose the wanted number of modes and to separately control their properties. Furthermore, each modal object has a number of pickup points, from which the sound is output. There must be at least one pickup point but they must be less than the number of modes. The first pickup point is also where the contact takes place (interaction point).

**inertial object** - simulates a simple inertial point mass. Obviously this kind of objects is useful solely as an exciter for other resonators. The only settable object property is its mass.

**waveguide object** - the digital waveguide technique [4] models the propagation of waves along elastic media. In the one-dimensional case implemented here [2], the waveguide object models an ideal elastic string. In our implementation it is possible to set length, tension and mass of the string. Further, one can set the position of the interaction point along its length. The interaction point coincides with the only available pickup point (i.e. the place from where the sound is output).

**Generalities**

Having a look at Fig. [1], the way two objects interact through an interactor appears evident: at each discrete time instant (sample) both objects send their internal states (displacement and velocity at the interaction point) to the interactor, which in turn sends the newly computed (opposite) forces to the objects. Knowing the new applied forces, the objects are able to compute their new states for the next time instant. In other words, there’s a feedback communication between the three models.

The SDT framework differs remarkably from the approach to physically-based sound synthesis found in most existing implementations and literature. Being this not the seat for an exhaustive and analytical dissertation and comparison, of interest is the IRCAM software Modalys [1], a powerful physical modeling interactive tool for musical applications, based on modal synthesis.
The Modalys working space is characterized by a modular set of modal objects, such as tubes, membranes, strings, “two-mass” objects and hybrids; types of linear connections, that is types of interactions between objects; controllers, that specify exactly how the connections will be executed; “accesses”, that set the physical location of the connection on a Modalys object in order to interact with other objects.

Recently, Modalys developments included the possibility to create three dimensional meshes and to compute their modes by finite elements numerical methods.

Available classes of modal objects include the following opcodes: “bi-string”, a string or rod whose vibration moves in two transverse directions; “cello-bridge”; “circ-membrane”, a circular membrane with zero thickness; “clamped-circ-membrane”, a circular plate fixed at its edges; “closed-closed-tube”, an acoustic tube sealed at both ends; “closed-open-tube”, an acoustic tube sealed at one end and opened at the other; “melt-hybrid” and “mix-hybrid” that create a hybrid of two different objects. The latter can be seen as a box with the two objects inside, with a sound mix of the two objects, whose excitation transmits energy to the sub-objects in proportion to the current position of the hybrid.

Connections-interactions can be modularly set on each object. Among the others, the relative opcodes includes: “adhere”, adherence between two accesses; “bi-fingerboard”, that simulates the interaction between a finger and a string with a fingerboard underneath; “bow”, a two dimensional connection between four accesses; “hole”, that makes a hole of variable diameter in an acoustic tube; “pluck”, where one access plucks another one; “position”, for changing the access position along its axis. At last, controllers include noise generators, break-point envelope functions, filters, oscillators and arithmetical operators.

It can be noticed how types of objects and interactions already form parametrically high level subclasses to be connected in order to achieve more complex virtual musical instruments.

As far as the interaction structures are concerned, a main difference between the SDT package and Modalys resides in the feed-forward structure of the latter: in Modalys a form exciter ↔ resonator is adopted, where non-linearities are concentrated in the interaction part of a structure (e.g. blow, bow, etc.).

Bearing in mind the musical purposes conception of Modalys, elements like key noise for an acoustic tube are sometimes simulated by means of a usual random generator, rather than actually being calculated as a physical result of other stimuli.

On the contrary, the SDT package takes advantage of a cartoonified approach in sound design and implements a feedback network within the interaction object1 ↔ interactor ↔ object2, with non-linear characteristics of the interactor. This allows the accurate modeling of complex interactions (e.g friction) and to output the sound of both the interacting objects. Besides, the continuous feedback approach adopted into the SDT is memory consistent, that is the system takes record of each previous state, during the interaction and manipulation. Secondly, the SDT package implements low level models that are already capable of all possible issues and interactions, with an open and wide variety of sound possibilities, and a consistent physical behavior.

**Externals conventions**

According to the described framework, the naming of each external conforms to the general form interactor_object1_object2~.

The linear impact interactor is referred as linpact, the non-linear impact interactor as impact, while the friction interactor is referred as friction.

The modal and the inertial objects are respectively referred as modalb and inertialb, while the waveguide object is referred as wg. The only naming exception happens when two modal objects interact, in which case the whole _object1_object2 section becomes _2modalb.

---

1The ending b is just a reminder for developers which tells that the bilinear transformation has been used in order to discretize the objects’ continuous-time equations.
Notice that object1 is always either a modal or an inertial object. This is because those are the only objects which are able to move: as already said, an inertial object behaves as a point mass, while the first mode of a modal object1 is its inertial mode, that is the whole object behaves as a mass-spring-damper structure. In both cases it is therefore possible to make object1 move (e.g. imposing an external force or an initial velocity on it) so that it starts interacting with object2.

As modal objects have a variable number of modes and pickup points, when an external involves a modal object the arguments are of variable length and composition. Moreover, each pickup point corresponds to a signal outlet, therefore outlets of such externals are also in variable number.

In the following externals descriptions, elements which are in variable number are underlined. If applicable, each argument type is followed by the name of the element (interactor, object1, object2) it refers to.

Help patch conventions

The best way to understand how the supplied externals work is to have a look to their help patches. Just be sure to put them under Max/MSP /max-help directory to have access to them when invoking the help.

All help patches share a common structure (cfr. Fig. 1.2): the control panels on the left and right sides (orange backgrounds) are respectively for object1 and object2, those in the center (green background) are for the interactor.

Control panels for object1 only operate on a subset of its own properties (basically for screen space’s sake), yet they allow to trigger the contact with object2. Conversely, the control panels provided for the interactor and object2 operate on the whole set of their available physical-geometric properties.

Figure 1.2: Solids contact example (help) patch
1.1.1 linpact_inertialb_modalb~

Linear impact between one inertial and one modal object.

Input

- **signal object1** 1st inlet. External force applied to **object1**.
- **signal object2** 2nd inlet. External force applied to **object2**.
- **signal** 3rd inlet. Additional displacement offset between the objects.
- **float object1** 4th inlet. Initial strike velocity.
- **list: [3×float]** 5th inlet. The first two arguments are respectively the **interactor**’s force stiffness and damping coefficient. The third is the mass of **object1**.
- **list: [3×float] object2** 6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
- **list: [float] object2** 7th inlet. As many arguments as the number of modes. Frequency of each mode.
- **list: [float] object2** 8th inlet. As many arguments as the number of modes. Decay time of each mode.
- **list: [int float] object2** 9th inlet. The **int** value indicates the considered pickup point. The subsequent list of **float** values sets the gain of each mode at the specified pickup point.
- **strike object1** 1st inlet. Followed by a **float** value, sets the initial strike velocity.
- **interact_strikr** 1st inlet. Followed by three **float** values. The first two set respectively the **interactor**’s force stiffness and damping coefficient. The third sets the mass of **object1**.
- **actmodes2 object2** 1st inlet. Followed by one **int** value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.
- **base2 object2** 1st inlet. Followed by three **float** values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
- **mode_freqs2 object2** 1st inlet. Followed by as many **float** values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the **base** message.
mode ts2 object2 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs2 object2 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

Output

All outlets come from pickup points of object2, in progressive order from left to right.

signal object2 As many signal outlets as the number of pickup points. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

list: [3×float] The first two arguments are respectively the interactor’s force stiffness and damping coefficient. The third is the mass of object1.

int object2 Number of modes.

int object2 Number of pickup points.

symbol object2 As many arguments as the number of pickup points. Mask for the output of each pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.

list: [3×float] object2 Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

list: [float] object2 As many arguments as the number of modes. Frequency of each mode.

list: [float] object2 As many arguments as the number of modes. Decay time of each mode.

list: [int float] object2 As many lists as the number of pickup points. The length of each list is equal to the number of modes +1. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.
1.1.2 [linpact\_2modalb~]

Linear impact between two modal objects.

**Input**

- **signal** object1 1st inlet. External force applied to object1.
- **signal** object2 2nd inlet. External force applied to object2.
- **signal** 3rd inlet. Additional displacement offset between the objects.
- **float** object1 4th inlet. Initial strike velocity.
- **list**: [2×float] interactor 5th inlet. The two arguments are respectively the force stiffness and the damping coefficient.
- **list**: [3×float] object1 6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
- **list**: [float] object1 7th inlet. As many arguments as the number of modes. Frequency of each mode.
- **list**: [float] object1 8th inlet. As many arguments as the number of modes. Decay time of each mode.
- **list**: [int float] object1 9th inlet. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.
- **list**: [3×float] object2 10th inlet. Same as for the 6th inlet.
- **list**: [float] object2 11th inlet. Same as for the 7th inlet.
- **list**: [float] object2 12th inlet. Same as for the 8th inlet.
- **list**: [int float] object2 13th inlet. Same as for the 9th inlet.
- **strike** object1 1st inlet. Followed by a float value, sets the initial strike velocity.
- **interact** interactor 1st inlet. Followed by two float values which set respectively the interactor's force stiffness and damping coefficient.
- **actmodes1** object1 1st inlet. Followed by one int value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.
- **base1** object1 1st inlet. Followed by three float values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
mode_freqs1 object1 1st inlet. Followed by as many float values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the base message.

mode_ts1 object1 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs1 object1 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

actmodes2 object2 1st inlet. Same as actmodes1.

base2 object2 1st inlet. Same as base1.

mode_freqs2 object2 1st inlet. Same as mode_freqs1.

mode_ts2 object2 1st inlet. Same as mode_ts1.

mode_contribs2 object2 1st inlet. Same as mode_contribs1.

Output

Outlets come from pickup points of both object1 and object2 (starting from left). For example, considering that object1 has one pickup and object2 has three, the first outlet would come from pickup0 of object1 and the subsequent ones respectively from pickup0, pickup1 and pickup2 of object2.

signal object1 As many signal outlets as the number of pickup points of object1. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

signal object2 Same as for object1.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

list: [2×float] interactor The two arguments are respectively the force stiffness and the damping coefficient.

int object1 Number of modes.

int object1 Number of pickup points.
symbol object1  As many arguments as the number of pickup points. 
Mask for the output of each pickup point: ‘1’ sets the pickup point
to output the object’s velocity, while anything else (e.g. ‘d’) sets
it to output the object’s displacement.

int object2  Number of modes.

int object2  Number of pickup points.

symbol object2  As many arguments as the number of pickup points. 
Mask for the output of each pickup point: ‘1’ sets the pickup point
to output the object’s velocity, while anything else (e.g. ‘d’) sets
it to output the object’s displacement.

list: [3×float] object1 Base (global) factors. These multiply respectively: the frequency,
the decay time and the gain of all modes.

list: [float] object1  As many arguments as the number of modes.
Frequency of each mode.

list: [float] object1  As many arguments as the number of modes.
Decay time of each mode.

list: [int float] object1  As many lists as the number of pickup points. The length of each
list is equal to the number of modes +1.
The int value indicates the considered pickup point. The subse-
quent list of float values sets the gain of each mode at the specified
pickup point.

list: [3×float] object2  Base (global) factors. These multiply respectively: the frequency,
the decay time and the gain of all modes.

list: [float] object2  As many arguments as the number of modes.
Frequency of each mode.

list: [float] object2  As many arguments as the number of modes.
Decay time of each mode.

list: [int float] object2  As many lists as the number of pickup points. The length of each
list is equal to the number of modes +1.
The int value indicates the considered pickup point. The subse-
quent list of float values sets the gain of each mode at the specified
pickup point.
1.1.3 \texttt{impact\_inertialb\_modalb\~}

Non-linear impact between one inertial and one modal object.

\textbf{Input}

\begin{itemize}
  \item \texttt{signal object1} \hspace{1cm} 1st inlet. External force applied to \texttt{object1}.
  \item \texttt{signal object2} \hspace{1cm} 2nd inlet. External force applied to \texttt{object2}.
  \item \texttt{signal} \hspace{1cm} 3rd inlet. Additional displacement offset between the objects.
  \item \texttt{float object1} \hspace{1cm} 4th inlet. Initial strike velocity.
  \item \texttt{list: \([4 \times \text{float}]\)} \hspace{1cm} 5th inlet. The first three arguments are for the \texttt{interactor}. They are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient. The fourth is the mass of \texttt{object1}.
  \item \texttt{list: \([3 \times \text{float}]\)} \hspace{1cm} 6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
  \item \texttt{list: \([\text{float}]\)} \hspace{1cm} 7th inlet. As many arguments as the number of modes. Frequency of each mode.
  \item \texttt{list: \([\text{float}]\)} \hspace{1cm} 8th inlet. As many arguments as the number of modes. Decay time of each mode.
  \item \texttt{list: \([\text{int float}]\)} \hspace{1cm} 9th inlet. The \texttt{int} value indicates the considered pickup point. The subsequent list of \texttt{float} values sets the gain of each mode at the specified pickup point.
  \item \texttt{strike object1} \hspace{1cm} 1st inlet. Followed by a \texttt{float} value, sets the initial strike velocity.
  \item \texttt{interact\_strikr} \hspace{1cm} 1st inlet. Followed by four \texttt{float} values. The first three set respectively the \texttt{interactor}’s force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient. The fourth sets the mass of \texttt{object1}.
  \item \texttt{actmodes2 object2} \hspace{1cm} 1st inlet. Followed by one \texttt{int} value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.
  \item \texttt{base2 object2} \hspace{1cm} 1st inlet. Followed by three \texttt{float} values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
  \item \texttt{mode\_freqs2 object2} \hspace{1cm} 1st inlet. Followed by as many \texttt{float} values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the \texttt{base} message.
\end{itemize}
mode_ts2 object2 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs2 object2 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

Output

All outlets come from pickup points of object2, in progressive order from left to right.

signal object2 As many signal outlets as the number of pickup points. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

list: [4×float] object2 The first three arguments are for the interactor. They are respectively the force stiffness, a parameter which depends on the contact surface’s shape, and the dissipation coefficient. The fourth is the mass of object1.

int object2 Number of modes.

int object2 Number of pickup points.

symbol object2 As many arguments as the number of pickup points. Mask for the output of each pickup point: ‘1’ sets the pickup point to output the object’s velocity, while anything else (e.g. ‘d’) sets it to output the object’s displacement.

list: [3×float] object2 Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

list: [float] object2 As many arguments as the number of modes. Frequency of each mode.

list: [float] object2 As many arguments as the number of modes. Decay time of each mode.

list: [int float] object2 As many lists as the number of pickup points. The length of each list is equal to the number of modes +1. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.
1.1.4 \texttt{impact2modalb~}

Non-linear impact between two modal objects.

**Input**

\begin{itemize}
  \item \texttt{signal object1} 1st inlet. External force applied to \texttt{object1}.
  \item \texttt{signal object2} 2nd inlet. External force applied to \texttt{object2}.
  \item \texttt{signal} 3rd inlet. Additional displacement offset between the objects.
  \item \texttt{float object1} 4th inlet. Initial strike velocity.
  \item \texttt{list: \{3×float\} interactor} 5th inlet. The three arguments are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.
  \item \texttt{list: \{3×float\} object1} 6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
  \item \texttt{list: \{float\} object1} 7th inlet. As many arguments as the number of modes. Frequency of each mode.
  \item \texttt{list: \{float\} object1} 8th inlet. As many arguments as the number of modes. Decay time of each mode.
  \item \texttt{list: \{int float\} object1} 9th inlet. The \texttt{int} value indicates the considered pickup point. The subsequent list of \texttt{float} values sets the gain of each mode at the specified pickup point.
  \item \texttt{list: \{3×float\} object2} 10th inlet. Same as for the 6th inlet.
  \item \texttt{list: \{float\} object2} 11th inlet. Same as for the 7th inlet.
  \item \texttt{list: \{float\} object2} 12th inlet. Same as for the 8th inlet.
  \item \texttt{list: \{int float\} object2} 13th inlet. Same as for the 9th inlet.
  \item \texttt{strike object1} 1st inlet. Followed by a \texttt{float} value, sets the initial strike velocity.
  \item \texttt{interact interactor} 1st inlet. Followed by three \texttt{float} values which set respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.
  \item \texttt{actmodes1 object1} 1st inlet. Followed by one \texttt{int} value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.
  \item \texttt{base1 object1} 1st inlet. Followed by three \texttt{float} values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
\end{itemize}
mode_freqs1 object1 1st inlet. Followed by as many float values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the base message.

mode_ts1 object1 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs1 object1 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

actmodes2 object2 1st inlet. Same as actmodes1.

base2 object2 1st inlet. Same as base1.

mode_freqs2 object2 1st inlet. Same as mode_freqs1.

mode_ts2 object2 1st inlet. Same as mode_ts1.

mode_contribs2 object2 1st inlet. Same as mode_contribs1.

Output

Outlets come from pickup points of both object1 and object2 (starting from left). For example, considering that object1 has one pickup and object2 has three, the first outlet would come from pickup0 of object1 and the subsequent ones respectively from pickup0, pickup1 and pickup2 of object2.

signal object1 As many signal outlets as the number of pickup points of object1. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

signal object2 Same as for object1.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

list: [3×float] interactor The three arguments are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.

int object1 Number of modes.

int object1 Number of pickup points.
symbol object1
As many arguments as the number of pickup points.
Mask for the output of each pickup point: '1' sets the pickup point
to output the object's velocity, while anything else (e.g. 'd') sets
it to output the object's displacement.

int object2
Number of modes.

int object2
Number of pickup points.

symbol object2
As many arguments as the number of pickup points.
Mask for the output of each pickup point: '1' sets the pickup point
to output the object's velocity, while anything else (e.g. 'd') sets
it to output the object's displacement.

list: [3×float] object1
Base (global) factors. These multiply respectively: the frequency,
the decay time and the gain of all modes.

list: [float] object1
As many arguments as the number of modes.
Frequency of each mode.

list: [float] object1
As many arguments as the number of modes.
Decay time of each mode.

list: [int float] object1
As many lists as the number of pickup points. The length of each
list is equal to the number of modes +1.
The int value indicates the considered pickup point. The subse-
quent list of float values sets the gain of each mode at the specified
pickup point.

list: [3×float] object2
Base (global) factors. These multiply respectively: the frequency,
the decay time and the gain of all modes.

list: [float] object2
As many arguments as the number of modes.
Frequency of each mode.

list: [float] object2
As many arguments as the number of modes.
Decay time of each mode.

list: [int float] object2
As many lists as the number of pickup points. The length of each
list is equal to the number of modes +1.
The int value indicates the considered pickup point. The subse-
quent list of float values sets the gain of each mode at the specified
pickup point.
Non-linear impact between one inertial and one waveguide object.

### Input

- **signal** object1: 1st inlet. External force applied to object1.
- **signal** object2: 2nd inlet. External force applied to object2.
- **signal** 3rd inlet: Additional displacement offset between the objects.
- **float** object1: 4th inlet. Initial strike velocity.
- **list**: [4×float] 5th inlet. The first three arguments are for the interactor. They are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient. The fourth is the mass of object1.
- **float** object2: 6th inlet. Normalized (0-1) contact position along string’s length.
- **float** object2: 7th inlet. String’s length in meters.
- **float** object2: 8th inlet. String’s tension in Newtons.
- **float** object2: 9th inlet. String’s mass in kilograms.
- **strike** object1: 1st inlet. Followed by a float value, sets the initial strike velocity.
- **interact_strikr** 1st inlet. Followed by four float values. The first three set respectively the interactor’s force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient. The fourth sets the mass of object1.
- **contact_pos2** object2: 1st inlet. Followed by a float value, sets the normalized (0-1) contact position along the string.
- **str_length2** object2: 1st inlet. Followed by a float value, sets the string’s length in meters.
- **str_tension2** object2: 1st inlet. Followed by a float value, sets the string’s tension in Newtons.
- **str_mass2** object2: 1st inlet. Followed by a float value, sets the string’s mass in kilograms.

### Output

The outlet comes from the only pickup point of object2.
Depending on the chosen output mask (see the Arguments section below), the outlet outputs either the object’s velocity or displacement at the contact position.

**Arguments**

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

- **list**: \([4 \times \text{float}]\) The first three arguments are for the interactor. They are respectively the force stiffness, a parameter which depends on the contact surface’s shape, and the dissipation coefficient. The fourth is the mass of \text{object1}.

- **float** \text{object2} Normalized (0-1) contact position along string’s length.

- **float** \text{object2} String’s length in meters.

- **float** \text{object2} String’s tension in Newtons.

- **float** \text{object2} String’s mass in kilograms.

- **symbol** \text{object2} Mask for the output of pickup point: ’1’ sets the pickup point to output the object’s velocity, while anything else (e.g. ’d’) sets it to output the object’s displacement.
Non-linear impact between one modal and one waveguide object.

**Input**

- **signal** object1 1st inlet. External force applied to object1.
- **signal** object2 2nd inlet. External force applied to object2.
- **signal** 3rd inlet. Additional displacement offset between the objects.
- **float** object1 4th inlet. Initial strike velocity.
- **list:** 
  - [3×**float**] interactor 5th inlet. The three arguments are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.
- **list:** 
  - [3×**float**] object1 6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
- **list:** 
  - [**float**] object1 7th inlet. As many arguments as the number of modes. Frequency of each mode.
- **list:** 
  - [**float**] object1 8th inlet. As many arguments as the number of modes. Decay time of each mode.
- **list:** 
  - [**int** **float**] object1 9th inlet. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.
- **float** object2 10th inlet. Normalized (0-1) contact position along string’s length.
- **float** object2 11th inlet. String’s length in meters.
- **float** object2 12th inlet. String’s tension in Newtons.
- **float** object2 13th inlet. String’s mass in kilograms.
- **strike** object1 1st inlet. Followed by a float value, sets the initial strike velocity.
- **interact** interactor 1st inlet. Followed by three float values which set respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.
- **actmodes1** object1 1st inlet. Followed by one int value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.
- **base1** object1 1st inlet. Followed by three float values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.
mode_freqs1 object1 1st inlet. Followed by as many float values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the base message.

mode_ts1 object1 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs1 object1 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

contact_pos2 object2 1st inlet. Followed by a float value, sets the normalized (0-1) contact position along the string.

str_length2 object2 1st inlet. Followed by a float value, sets the string’s length in meters.

str_tension2 object2 1st inlet. Followed by a float value, sets the string’s tension in Newtons.

str_mass2 object2 1st inlet. Followed by a float value, sets the string’s mass in kilograms.

Output

Outlets come from pickup points of both object1 and object2 (starting from left). The rightmost one comes from the only pickup point of object2.

signal object1 As many signal outlets as the number of pickup points of object1. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

signal object2 Depending on the chosen output mask (see the Arguments section below), the rightmost outlet outputs either the object’s velocity or displacement at the contact position.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

list: [3×float] interactor The three arguments are respectively the force stiffness, a parameter which depends on the contact-surface’s shape, and the dissipation coefficient.

int object1 Number of modes.
int object1  Number of pickup points.

symbol object1  As many arguments as the number of pickup points.
Mask for the output of each pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.

list: [3×float] object1  Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

list: [float] object1  As many arguments as the number of modes.
Frequency of each mode.

list: [float] object1  As many arguments as the number of modes.
Decay time of each mode.

list: [int float] object1  As many lists as the number of pickup points. The length of each list is equal to the number of modes +1.
The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.

float object2  Normalized (0-1) contact position along string’s length.

float object2  String’s length in meters.

float object2  String’s tension in Newtons.

float object2  String’s mass in kilograms.

symbol object2  Mask for the output of pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.
1.1.7 **摩擦_2modalb~**

Non-linear friction between two modal objects.

**Input**

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal</td>
<td>object1</td>
<td>1st inlet. External force applied to object1.</td>
</tr>
<tr>
<td>signal</td>
<td>object2</td>
<td>2nd inlet. External force applied to object2.</td>
</tr>
<tr>
<td>signal</td>
<td></td>
<td>3rd inlet. Additional displacement offset between the objects.</td>
</tr>
<tr>
<td>float</td>
<td>object1</td>
<td>4th inlet. Initial rubbing velocity.</td>
</tr>
<tr>
<td>list:</td>
<td>[9×float]</td>
<td>5th inlet. The nine arguments are respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Strubeck velocity, the perpendicular pressure which object1 applies on object2, and finally the noise intensity.</td>
</tr>
<tr>
<td>list:</td>
<td>[3×float]</td>
<td>6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.</td>
</tr>
<tr>
<td>list:</td>
<td>[float]</td>
<td>7th inlet. As many arguments as the number of modes. Frequency of each mode.</td>
</tr>
<tr>
<td>list:</td>
<td>[float]</td>
<td>8th inlet. As many arguments as the number of modes. Decay time of each mode.</td>
</tr>
<tr>
<td>list:</td>
<td>[int float]</td>
<td>9th inlet. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.</td>
</tr>
<tr>
<td>list:</td>
<td>[3×float]</td>
<td>10th inlet. Same as for the 6th inlet.</td>
</tr>
<tr>
<td>list:</td>
<td>[float]</td>
<td>11th inlet. Same as for the 7th inlet.</td>
</tr>
<tr>
<td>list:</td>
<td>[float]</td>
<td>12th inlet. Same as for the 8th inlet.</td>
</tr>
<tr>
<td>list:</td>
<td>[int float]</td>
<td>13th inlet. Same as for the 9th inlet.</td>
</tr>
<tr>
<td>start_rubbing</td>
<td>object1</td>
<td>1st inlet. Followed by a float value, sets the initial rubbing velocity.</td>
</tr>
<tr>
<td>interact_pressr</td>
<td>object1</td>
<td>1st inlet. Followed by nine float values which set respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Strubeck velocity, the perpendicular pressure which object1 applies on object2, and finally the noise intensity.</td>
</tr>
</tbody>
</table>
actmodes1 object1 1st inlet. Followed by one int value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.

dbase1 object1 1st inlet. Followed by three float values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

mode_freqs1 object1 1st inlet. Followed by as many float values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the base message.

mode_ts1 object1 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs1 object1 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

actmodes2 object2 1st inlet. Same as actmodes1.

dbase2 object2 1st inlet. Same as base1.

mode_freqs2 object2 1st inlet. Same as mode_freqs1.

mode_ts2 object2 1st inlet. Same as mode_ts1.

mode_contribs2 object2 1st inlet. Same as mode_contribs1.

Output

Outlets come from pickup points of both object1 and object2 (starting from left). For example, considering that object1 has one pickup and object2 has three, the first outlet would come from pickup0 of object1 and the subsequent ones respectively from pickup0, pickup1 and pickup2 of object2.

signal object1 As many signal outlets as the number of pickup points of object1. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

signal object2 Same as for object1.

Arguments

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:
The nine arguments are respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Stribeck velocity, the perpendicular pressure which object1 applies on object2, and finally the noise intensity.

**list: [9×float]**

The nine arguments are respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Stribeck velocity, the perpendicular pressure which object1 applies on object2, and finally the noise intensity.

**int** object1 Number of modes.

**int** object1 Number of pickup points.

**symbol** object1 As many arguments as the number of pickup points. Mask for the output of each pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.

**int** object2 Number of modes.

**int** object2 Number of pickup points.

**symbol** object2 As many arguments as the number of pickup points. Mask for the output of each pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.

**list: [3×float]** object1 Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

**list: [float]** object1 As many arguments as the number of modes. Frequency of each mode.

**list: [float]** object1 As many arguments as the number of modes. Decay time of each mode.

**list: [int float]** object1 As many lists as the number of pickup points. The length of each list is equal to the number of modes +1. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.

**list: [3×float]** object2 Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

**list: [float]** object2 As many arguments as the number of modes. Frequency of each mode.

**list: [float]** object2 As many arguments as the number of modes. Decay time of each mode.
As many lists as the number of pickup points. The length of each list is equal to the number of modes +1. The `int` value indicates the considered pickup point. The subsequent list of `float` values sets the gain of each mode at the specified pickup point.
**1.1.8 friction_modalb_wg~**

Non-linear friction between one modal and one waveguide object.

**Input**

- **signal** `object1` 1st inlet. External force applied to `object1`.
- **signal** `object2` 2nd inlet. External force applied to `object2`.
- **signal** Additional displacement offset between the objects.
- **float** `object1` 4th inlet. Initial bowing velocity.

**list: [9×float]**

5th inlet. The nine arguments are respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Stribeck velocity, the perpendicular pressure which `object1` applies on `object2`, and finally the noise intensity.

**list: [3×float]**

6th inlet. Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

**list: [float]**

7th inlet. As many arguments as the number of modes. Frequency of each mode.

**list: [float]**

8th inlet. As many arguments as the number of modes. Decay time of each mode.

**list: [int float]**

9th inlet. The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.

- **float** `object2` 10th inlet. Normalized (0-1) contact position along string’s length.
- **float** `object2` 11th inlet. String’s length in meters.
- **float** `object2` 12th inlet. String’s tension in Newtons.
- **float** `object2` 13th inlet. String’s mass in kilograms.

- **start_bowing** `object1` 1st inlet. Followed by a float value, sets the initial bowing velocity.

**interact_pressr**

1st inlet. Followed by nine float values which set respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Stribeck velocity, the perpendicular pressure which `object1` applies on `object2`, and finally the noise intensity.
actmodes1 object1 1st inlet. Followed by one int value which sets the number of currently active modes. Obviously the maximum number of active modes must be lower than the total number of available modes.

base1 object1 1st inlet. Followed by three float values correspondent to the base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

mode_freqs1 object1 1st inlet. Followed by as many float values as the number of modes which set each mode frequency. Notice that for this message to have effect you need to refresh the base message.

mode_ts1 object1 1st inlet. Followed by as many float values as the number of modes which set their decay times. Notice that for this message to have effect you need to refresh the base message.

mode_contribs1 object1 1st inlet. Followed by one int value which specifies a pickup point, and as many float values as the number of modes. These set the gain (0-100 on a logarithmic scale) of each mode at the specified pickup point.

contact_pos2 object2 1st inlet. Followed by a float value, sets the normalized (0-1) contact position along the string.

str_length2 object2 1st inlet. Followed by a float value, sets the string’s length in meters.

str_tension2 object2 1st inlet. Followed by a float value, sets the string’s tension in Newtons.

str_mass2 object2 1st inlet. Followed by a float value, sets the string’s mass in kilograms.

Output

Outlets come from pickup points of both object1 and object2 (starting from left). The rightmost one comes from the only pickup point of object2.

signal object1 As many signal outlets as the number of pickup points of object1. Depending on the chosen output mask (see the Arguments section below), they output either the object’s velocity or displacement.

signal object2 Depending on the chosen output mask (see the Arguments section below), the rightmost outlet outputs either the object’s velocity or displacement at the contact position.
**Arguments**

All arguments are mandatory. They initialize the two objects and the interactor with their physical-geometric properties. In progressive order they are:

- **list: \([9\times \text{float}]\)**
  - The nine arguments are respectively: the mean bristles-stiffness, the mean bristles-dissipation, a viscosity coefficient, the dynamic-friction coefficient, the static-friction coefficient, the break-away coefficient, the Stribeck velocity, the perpendicular pressure which \texttt{object1} applies on \texttt{object2}, and finally the noise intensity.

- **int \texttt{object1}**
  - Number of modes.

- **int \texttt{object1}**
  - Number of pickup points.

- **symbol \texttt{object1}**
  - As many arguments as the number of pickup points.
  - Mask for the output of each pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.

- **list: \([3\times \text{float}]\)**
  - Base (global) factors. These multiply respectively: the frequency, the decay time and the gain of all modes.

- **list: \([\text{float}]\)**
  - As many arguments as the number of modes.
  - Frequency of each mode.

- **list: \([\text{float}]\)**
  - As many arguments as the number of modes.
  - Decay time of each mode.

- **list: \([\text{int float}]\)**
  - As many lists as the number of pickup points. The length of each list is equal to the number of modes +1.
  - The int value indicates the considered pickup point. The subsequent list of float values sets the gain of each mode at the specified pickup point.

- **float \texttt{object2}**
  - Normalized (0-1) contact position along string’s length.

- **float \texttt{object2}**
  - String’s length in meters.

- **float \texttt{object2}**
  - String’s tension in Newtons.

- **float \texttt{object2}**
  - String’s mass in kilograms.

- **symbol \texttt{object2}**
  - Mask for the output of pickup point: '1' sets the pickup point to output the object’s velocity, while anything else (e.g. 'd') sets it to output the object’s displacement.
1.2 Liquids

Low level models of basic events responsible for acoustic emission in liquids are considered. In particular, we consider the formation of single resonating cavities (bubbles) observed in dripping, boiling, or pouring.

Bubble model

The formation of radially oscillating bubbles under the surface of a liquid volume is modeled assuming that the bubble cavity acts as a simple Helmholtz resonator, its impulse response being a damped sinusoid with time-varying frequency \[5\]. One of the most common events involving the formation of radially oscillating bubbles under the surface of a liquid volume is dripping, the falling of a drop or an object into a quiescent liquid. In the simplest case, when the initial impact sound is neglected and when there is no crown formation following the initial impact, dripping can be represented by a single bubble event.

However, the more the initial impact sound is perceivable and the cavity of the bubble becomes larger (e.g., for large impacting mass), the less the simple single bubble sound model becomes adequate to represent the dripping event. This is even more evident when large objects or drops falling into a resting liquid generate many secondary bubbles and droplets events due to the mass displaced by the principal impact event (splashing).

The external onebubble~ provides controls for the bubble radius, and for the slope of the frequency rise due to radius change in time.

![Figure 1.3: Single bubble help patch](image-url)
1.2.1 onebubble~

Single radially oscillating bubble.

**Input**

- **bang**: 1st inlet. Bubble event trigger.
- **float**: 2nd inlet. Bubble initial radius.
- **float**: 2nd inlet. Frequency slope.
- **radius**: 1st inlet. Followed by a float value, sets the initial bubble radius.
- **slope**: 1st inlet. Followed by a float value, sets the frequency slope.

**Output**

- Radiated pressure.
- **signal**: Signal outlet.

**Arguments**

None.
2 Higher-level models

This chapter covers algorithms which exploit the low-level models seen in Chapter 1. Notice that the expression “higher-level” indicates more complex and structured algorithms, corresponding to somewhat large-scale events, processes or textures. In a way, that matches the meaning of the expression “high-level” in Computer Science, where it often denotes languages similar to those of human beings. Of course, in order to achieve that, high-level languages are indeed more complex and structured than low-level ones.

The higher-level algorithms here discussed implement temporal patterns or other physically consistent controls (e.g. external forces) superimposed to low-level models. These algorithms are made available either by dedicated externals, or by using Max/MSP visual programming, that is as patches. In the former case, the external can be self-contained - that is it implements both the control and the low-level synthesis layers - or just meant to drive other externals (in which case it is labeled with a final tilde ~) - that is it implements only the control layer (in which case it is labeled without a final tilde ~).

2.1 Solids

2.1.1 Bouncing

Bouncing is a sound process resulting from repeated macro-impact events.

Figure 2.1: Bouncing (falling object) patch
The Max/MSP patch (BOUNCING.mxb) (Fig. 2.1) simulates the case of a falling object - a typical situation in which bouncing happens. There, an instance of the low-level external impact inertialb modalb ~ is driven by a control layer implemented into the sub-patch dropper (which lies inside the sub-patch sub-bouncing).

The available high-level user's controls are:

**falling height:** sets the time interval between the first two bounces.

**object elasticity:** sets the acceleration and deceleration rate of bounces.

**object shape regularity:** sets the level of randomness to be applied to bounces. This corresponds to the symmetry properties of the falling object.

**object weight:** sets the maximum hitting velocity.
2.1.2 control_crump

Crumpling is a widely general-purpose sonification paradigm based on the control of atomic sonic events (for instance impacts). The crumpling process governs the timbre and dynamics of every atomic event as well as the temporal gap in between two successive events.

- Concerning dynamics and time, their control is exerted by statically setting macroscopic parameters of specific stochastic laws which, in their turn, produce sequences of micro events whose overall intensity and temporal density follow directly from the parameter values. However, every sequence individually exhibits statistical variability.
- Concerning timbre, its control follows by informing the model with microscopic features (viz. the nature of the atomic sounds) along with changing (usually coarse) structural evolutionary features of the physical model dynamically along time.

Detailed explanations about the physical nature of this sonification paradigm, in particular the correspondence existing between an atomic crumpling event and its representation as an impact between a hammering and a resonating object, can be found in [3].

Figure 2.2: Crumpling example patch

The external control_crump implements such controls and is expressly meant to drive impact events modeled by two instances of impact_inertialb_modalb~ externals in which both object2s (modal objects) have two resonating modes. Hence it controls two couples of interacting objects (left: object1 and object2, right object1 and object2). In Fig. 2.2 is represented the provided example patch (CRUMPLING.mxb) in which control_crump drives the low-level sound synthesis models.

Input

- **bang** 1st inlet. A new atomic crumpling event is triggered.
- **reset** 1st inlet. The crumpling process is reset by restoring initial energy and settings.
float 2nd inlet. Size of the crumpling. Initializes the energy.

float 3rd inlet. Force of the crumpling.

float 4th inlet. Smoothness of the crumpling.

list: [2×float] 5th inlet. Sets the range in which the frequency-factors (multiplying coefficients for resonances) of both object2s vary along the duration of crumpling process. The two float arguments are respectively the initial and the final values of the coefficients.

list: [4×float] 6th inlet. Sets the range in which the decay times of the two modes of both object2s vary along the duration of crumpling process. The four float arguments are, in order: the initial and the final values of the decay for the first mode of both object2s; the initial and the final values of the decay for the second mode of both object2s.

list: [2×float] 7th inlet. Sets the range in which the masses of both object1s vary along the duration of crumpling process. The two float arguments are the initial and the final values of the masses.

Output

Outlets provide controls to two instances of impact_inertialb.modalb~ externals, where both object2s have two resonating modes.

float 1st outlet. Delay value after which to back-bang the control_crump external itself by means of a loopback containing a delay object.

float 2nd outlet. Mass of the left object1.

float 3rd outlet. Frequency-factor of the left object2.

float 4th outlet. Decay value of the left object2's first mode.

float 5th outlet. Decay value of the left object2's second mode.

float 6th outlet. Energy of the signal outcoming from the left object2.

float 7th outlet. Mass of the right object1.

float 8th outlet. Frequency-factor of the right object2.

float 9th outlet. Decay value of the right object2's first mode.

float 10th outlet. Decay value of the right object2's second mode.

float 11th outlet. Energy of the signal outcoming from the right object2.
Arguments

All arguments are mandatory.

**float**  
Crumpling size.

**float**  
Crumpling force.

**float**  
Crumpling smoothness.

**list: [2×float]**  
Default range (initial and final values) of the frequency-factors of both object2s.

**list: [4×float]**  
Default range of the decay times. The four float arguments are, in order: the initial and the final values of the decay for the first mode of both object2s; the initial and the final values of the decay for the second mode of both object2s.

**list: [2×float]**  
Default range (initial and final values) of the mass of the both object1s.

**list: [2×float]**  
Cutoff frequency sweep range (initial and final values). It is meant to be connected to a lowpass filter with controllable cutoff frequency.
2.2 Liquids

High level models of two classes of complex liquid events are considered: 1. the formation of high quantities of bubbles observed in boiling, frying, streaming or pouring, and 2. the temporal pattern involving initial impacts, principal bubble formation, and secondary droplets, occurring in splash-like events.

Boiling, frying, streaming, pouring

The single resonating bubble model is well suited to serve as the elementary brick to represent populations of bubbles, whose parameters and triggering instants are designed according to given statistical distributions. Typical phenomena in which bubble distributions are observed are boiling or frying liquids, water streaming, pouring of a liquid into a container or into another quiescent liquid, breaking waves.

The external bubblestream provides controls for bubble triggering frequency, for the mean and variance of radius and radius slope values, for the variance of bubbles amplitude, and for the smoothness of bubbles onset. A sample patch is provided which contains presets for boiling, frying, and running water sounds.

![Figure 2.3: Bubblestream patch](image)
2.2.1 \texttt{bubblestream~}

Population of bubbles.

\textbf{Input}

- \texttt{bang} 1st inlet. Toggles the bubbles flow.
- \texttt{float} 2nd inlet. Mean bubbles radius.
- \texttt{float} 3rd inlet. Variance of bubbles radius.
- \texttt{float} 4th inlet. Mean radius slope.
- \texttt{float} 5th inlet. Variance of radius slope.
- \texttt{float} 6th inlet. Mean amplitude.
- \texttt{float} 7th inlet. Bubbles onset smoothness.
- \texttt{float} 8th inlet. Bubbles frequency.

\textbf{Output}

Radiated sound pressure.

\textbf{Arguments}

None.
Dripping, splashing

As an attempt to reproduce the temporal structure of a splashing sound, we choose to design it as a sequence of three distinct low-level events: 1. a short initial impact sound, 2. a bubble sound modeled as detailed in the low-level models section, and 3. a secondary droplets event texture. In the present implementation, sampled waveforms are used to reproduce the initial impact sound and the final sound due to droplets formation, whereas the principal bubble formation is based on the single bubble model.

The external splash~ provides controls for gain balance of the single components, and for the parameters of the principal bubble sound. A sample patch is provided which contains presets for single bubble and complete splashing sounds.

Figure 2.4: Splash patch
2.2.2 splash~

Splash or dripping event.

**Input**

- **bang**: 1st inlet. Splash event trigger.
- **float**: 2nd inlet. Principal bubble initial radius.
- **float**: 2nd inlet. Radius slope.
- **float**: 3d inlet. Initial impact event gain.
- **float**: 4th inlet. Principal bubble gain.
- **float**: 5th inlet. Droplet event gain.

**Output**

Radiated pressure.

**Arguments**

None.
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